Photonic Bell states creation around rotating black holes

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We argue that spinning black holes are capable to implement complex quantum information processes encoded in X-ray photons emitted by the accretion disk. Recently, numerical simulations showed that X-ray photons emitted by accretion disk acquire rotation of polarization angle and orbital angular momentum due to strong gravitational field in the vicinity of the rotating black holes. Based on these two degrees of freedom we construct a bipartite two-level quantum system of the accretion disks photons. To characterize the quantum states of this system we consider linear entropy for the reduced density matrix of polarization with the intention to exploit its direct relation with the photons degree of polarization. Since the X-ray radiation has a minimum degree of polarization located at the transition region of the accretion disk, the linear entropy is higher for the photons emitted on this region inferring a higher degree of entanglement for the composite system. We emphasize that for an extreme rotating black hole in the thermal state, the photons with energies at the thermal peak are maximally entangled in polarization and orbital angular momentum, leading to the creation of all four Bell states. Detection of the Bell states encoded by X-ray photons emitted nearby rotating black holes should be possible by equipment actually used in quantum information processing.

I.

Rotating black holes (RBH) are among the most mysterious predictions general relativity has made. Spinning black holes are literally dragging and twisting the space-time nearby influencing the photons emitted by the accretion disk. The strong gravitational field in the vicinity of spinning BH rotates the angle of polarization and imprint orbital angular momentum to X-ray photons emitted by accretion disk.

The efforts to determine the characteristics of these remarkable astrophysical bodies were mainly focused in the recent years over detecting the polarization of X-ray radiation coming from the accretion disk. It was pointed out in [1], [2], [3], [4], [5], [6] based on Stokes parameters calculation, that photons emitted by the accretion disk of black holes in the thermal state, should possess linear polarization, either parallel or perpendicular to the plane of the disk.

On the outer region of the disk, at low energies the X-ray radiation is horizontally polarized parallel to the plane of the disk. The polarization angle of photons coming from the innermost region is shifted through vertical polarization, perpendicular to the disk plane due to the strong gravitational field in the vicinity of the RBH.In this region, at high energies the X-ray photons are vertically polarized. The degree of polarization is maximal in these two regions of the accretion disk. Between the outer and inner regions of the disk, at the transition region the relative contributions of horizontal and vertical polarized photons are nearly equal and no net polarization is observed. The transition region is characterized by very low degrees of polarization that tends to zero for near-extreme spinning black holes.

On the other hand, numerical simulations [10], [11], [12], [13] suggest that strong dragging frame effect near rotating black holes imprint nontrivial orbital angular momentum (OAM) modes to photons emitted by the accretion disk. X-ray photons coming from the disk are forced by the twisted spacetime around spinning black hole to acquire OAM. As the BH spins faster the OAM modes spectrum extend to wider values carrying both positive and negative OAM values ($\pm\ell$).

When measured by a distant observer, the X-ray radiation coming from accretion disk should carry linear polarization (horizontal or vertical) and specific (positive or negative) OAM values as a consequence of spacetime shape modeled by the presence of rotating black hole.

To completely determine the states of photons emitted near rotating black holes we consider the density matrix [7], [8] of the composite system consisting of the two degrees of freedom - polarization and OAM which form Hermitian and orthogonal two-level subsystems.

To simplify further the analysis of this bipartite system consisting of two-level subsystems, polarization and OAM, we use the reduced density matrix of photons polarization having in mind to exploit its direct relation with the photon degree of polarization via the linear entropy. Our main assumption here is that the degree of mixed-ness of the polarization subsystem determines the degree of entanglement of the composite system, polarization-OAM as in [14], [15], [16], [17], [18], [19]. The high mixed-ness of the subsystem suggests a strong non-separability (entanglement) of the composite system.

We consider linear entropy of the X-ray photons polarization in order to shed light on the degree of mixedness in this subsystem. Broadly, the low degree of polarization of X-ray radiation on the transition region of the accretion disk is an indication of high values of linear polarization and so a high degree of mixed-ness at the subsystem level; an expected result since on the transition region both horizontal and vertical polarized photons are present. From here we conclude that X-ray photons emitted by accretion disk possess a certain degree of entanglement in polarization and OAM modes.

The outer and innermost regions of rotating black hole accretion disk emit photons closer to pure state, due to the strong horizontal and, respectively vertical polarization. The X-ray photons emitted on the transition region are in mixed states which suggest entangled to a certain degree that depends on

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BH spin. The faster the BH spins the higher de degree of entanglement of X-ray photons in polarization and OAM.

The maximal entangled state is realized in the case of an extreme spinning BH ($a = 1$), for the photons at the energy peak in the transition region.

The maximal entanglement in polarization and OAM of X-ray photons coming from the transition region of the accretion disk is expressed via Schmidt decomposition [26], [27] by the all four Bell states.

The Bell states of X-ray photons can be measured by simple quantum optics set ups [20], [21] with the limitation imposed here by the high energy of photons (X-ray band). With the later progress in technology related to X-ray laboratory research [23], [24], [25] leading to quantum computation with X-ray photons [22] legitimate hopes are directed towards detection and measurements of the Bell states the X-ray radiation coming from galactic active nuclei and solar-mass black holes accretion disk.

Detection of Bell states is an important indication on the possibility that strong gravitational field near rotating black holes to literally implement complex quantum information processes encoded in the X-ray photons emitted by accretion disk.

II.

Expectations to unveil some of the black holes hidden characteristics, such as the speed of spinning, are related to measuring the polarization of X-ray photons emitted by the accretion disk. Polarization of radiation emitted by accretion disk in thermal state is analyzed in [1] [2], [4], [5] considering the simplest model of a geometrically thin, optically thick, steady-state accretion disk, aligned with the BH spin axis.

Stokes parameters are central in attempts [1], [2], [3], [4], [5], [6] to estimate the degree of polarization and the polarization angle of X-ray radiation emitted by accretion disk near RBH:

$$s_0 = I \tag{1}$$

$$s_1 = Q = Icos2\beta cos2\chi \tag{2}$$

$$s_2 = U = Icos2\beta sin2\chi \tag{3}$$

$$s_3 = V = Isin2\beta \tag{4}$$

where χ is the polarization angle.

Angle of polarization and degree of polarization, the two parameters of interest that quote the characteristics of black hole are derived from the Stokes parameter in the following manner :

$$tan2\chi = \frac{U}{Q} \tag{5}$$

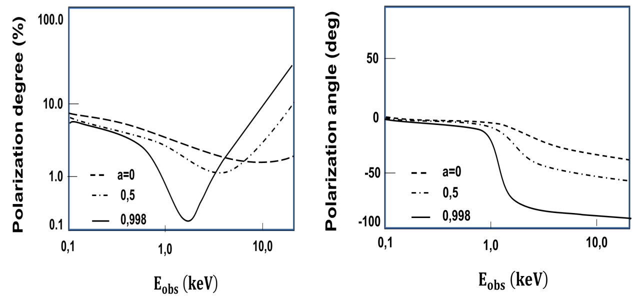

Figure 1. Polarization degree (left) and polarization angle for X-ray photons emitted by accretion disk of BH with different spin parameter.

$$\delta = \sqrt{Q^2 + U^2} \tag{6}$$

we consider here only the linear polarization since the polarization is induced by Compton scattering. These two parameters are sensitive to the black hole spin (a) and the inclination angle (i) of the accretion disk in relation with a distant observer that detect X-ray photons coming from the accretion disk. The angle of inclination is not relevant here and will be assigned to a constant value $i = 45^0$; only the spin parameter of the black hole is inferred further.

Numerical simulations results, considering different spin parameters (a) are resumed in the figure 1, where the degree of polarization of X-ray radiation is depicted in left panel and the angle of polarization in the right image.

It can be noted from the left image in the figure 1 that X-rays emitted near the outer region of the accretion disk, at low energies, are polarized parallel to the plane of the disk, horizontal polarization. The strong gravitational field close to the black hole rotates the polarization angle of the X-ray photons emitted by the innermost regions of the disk to positive or negative values perpendicular to the plane of the disk; vertical polarization. The rotation of polarization angle is greater for rapidly spinning black holes where accretion disk moves closer to the horizon.

At the transition region of the accretion disk situated between low energies and high energies of X-ray radiation, both horizontal and vertical polarization photons are present canceling each other contribution, phenomenon reflected in a minimum peak of the degree of polarization. It can be noted from the figure 1 (left) that for near-extreme rotating black holes ($a = 0,998$) the degree of polarization of photons at the peak energies emitted in the transition regions goes to a minimum that almost approach zero value ($\delta = 0$). This is an important result which will play a crucial role in what follows.

Unlike radio or optical telescopes, which measure the intensity of the radiation from the source, most X-ray telescopes detect individual photons. Using the additivity of Stokes parameters and counting a large number of photons the above results are recovered. Since the number of X-ray photons is small we desire to extract as much information as is physically possible from every photon.

Stokes parameters could be inferred to construct the polarization coherency matrix of the photons emitted near rotating black holes, a less considered approach [7], [8] which unitary contain all physical information about the polarization state of radiation. The polarization coherency matrix is given by:

$$\rho_p = \frac{1}{2}\begin{bmatrix} I+Q & U-iV \\ U+iV & I-Q \end{bmatrix} \tag{7}$$

Replacing the values of Stokes parameters from the equations (1) ,polarization matrix takes the form:

$$\rho_p = \begin{bmatrix} 1+cos2\beta cos2\chi & cos2\beta sin2\chi - isin2\beta \\ cos2\beta sin2\chi + isin2\beta & 1-cos2\beta cos2\chi \end{bmatrix} \tag{8}$$

Only linear polarization of X-ray radiation coming from the accretion disk is retain further since the comptonization of radiation scattered from the disk prevent the circular polarization. Taking $\beta = 0$ and by straightforward calculations the coherency matrix could be written as:

$$\rho_p = \begin{bmatrix} cos^2\chi & cos\chi sin\chi \\ cos\chi sin\chi & sin^2\chi \end{bmatrix} \tag{9}$$

The coherency polarization matrix equals the density matrix of the photons source [14], [15] since density matrix is the unit trace scaling of the polarization matrix and $Tr(\rho_p) = 1$, such as for the rest of the present paper we will refer to ρ_p as the density matrix of polarization of X-ray photons.

The density matrix corresponds to the general quantum state of X-ray photons emitted by the accretion disk near the rotating black hole:

$$|\Phi_{pol}\rangle = cos\chi\,|H\rangle + sin\chi\,|V\rangle \tag{10}$$

where H and V are horizontal and respectively vertical polarization.

Considering that only linear polarization horizontal (H) and vertical (V) to the plane of the accretion disk is measured the density matrix reduces to:

$$\rho_p = \begin{bmatrix} cos^2\chi & 0 \\ 0 & sin^2\chi \end{bmatrix} \tag{11}$$

The matrix (7) is the diagonalized two dimensional Hermitian (H_{pol}) density matrix of the two-level orthogonal system of X-ray photons polarization, considered in the basis H, V as the degree of freedom.

The states of the X-ray photons are parts of the Hermitian space of polarization $|\Phi_{pol}\rangle \in H_{pol}$.

Based on the density matrix representation, we can now characterize the state of the X-ray photons emitted near the outside region of accretion disk, horizontally (H) polarized, the density matrix is $\rho_p = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, while the $\rho_p = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$.

At the transition region X-ray photons form a mixture of pure states consisting of photons polarized equally horizontal and vertical $\rho_p = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$.

The mixture of horizontal and vertical polarization of accretion disk X-ray photons is in accordance with the figure 1.

III.

Acquiring orbital angular momentum by photons emitted or passing nearby the equatorial plane of rotating black holes, the hypothesis formulated by Harwit [9], gained consistency lately in [10], [11], [12], [13].

Tamburini et al. [10] performed numerical simulations of the radiation emitted from an accretion disk of a rotating black hole and observed nontrivial photon OAM generation and asymmetric spectra in terms of the LG-mode. Radiation beam emitted by accretion disk acquiring independent azimuthal phase term $e^{i\ell\phi}$ possesses an orbital angular momentum of ℓ per photon, where ℓ is the integer topological charge.

The OAM values ℓ that the frame dragging effect nearby spinning black holes imprint to X-ray radiation emitted by accretion disk are determined by BH spin and the inclination angle of the disk towards a remote observer. We maintain for the inclination angle the same value $i = 45^0$ as discussed in the case of X-ray polarization and analyze further the only the BH spin parameter influences on the X-ray radiation coming from the accretion disk.

The figure 2 depicts the maps of OAM modes acquired by X-ray photons emitted near RBH having two different values for the spin parameter, a relatively slow spinning BH ($a = 0,5$) to the left and a near-extreme rotating BH ($a = 0,99$) to the right, captured from [10].

Notice from figure 2 that the slow spinning BH is characterized by a narrow spectrum of OAM that reduces only to zero LG mode ($\ell = 0$) for Schwartzchild BH ($a = 0$) , which is still the predominant value for spin parameter $a = 0,5$.

A rapidly spinning black hole ($a = 0,99$) has the innermost region of the accretion disk closer to the black hole event horizon. X-ray photons emitted by the accretion disk are influenced stronger by the gravitational field because of the BH proximal vicinity and tends to flip toward wider OAM modes. As it can be noticed from figure 2 (right) the near-extreme spinning Black hole has a broader OAM spectrum with a significant reduction of zero LG modes.

We are interested here of another remarkable aspect that reside from the OAM maps above; as can be inferred from figure 2, X-ray photons emitted near the RBH acquire both negative and positive OAM values. Considering the OAM modes, the state of X-ray photons can be expressed as a combination of positive and negative OAM modes, $-\ell$ and ℓ .

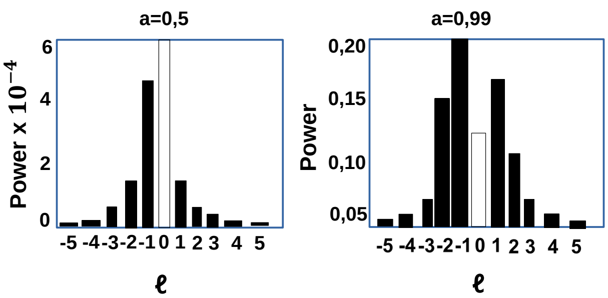

Figure 2. OAM modes spectrum for X-ray photons emitted by the accretion disk of two different spin parameter black holes slow spinning BH (left) and high spinning BH (right). Caption from [10].

The generic X-ray photon emitted nearby RBH acquires not only a definite polarization (horizontal or vertical) but also a definite value of OAM ($\pm\ell$). The OAM modes ($\pm\ell$) can possess theoretically any values in the interval $[-\infty,\infty]$. The photon OAM ($\pm\ell$) are orthogonal [28], [29] and form an unbound Hilbert space unlike the case of photon polarization which is two-dimensional. We restrict the OAM unbound Hilbert space to a two dimensional Hilbert(H_{OAM}) space by considering a generic subspace of OAM ($\pm\ell$), were ℓ can take precise values $\pm1, \pm2, ...$, as it will be seen later in this paper. For now we will consider generic OAM ($\pm\ell$) having in mind a specific value of X-ray photons OAM. To consistently specify the state of X-ray photons emitted by the accretion disk both of the two degrees of freedom must be considered. The polarized photon has now one more degree of freedom to assess its quantum state, the OAM mode, such as a photon horizontally/vertically (H/V) polarized also possess $-\ell$ or ℓ OAM modes. The state of photons emitted by the RBHs accretion disk can have one of the states $(H,\ell), (H,-\ell), (V,\ell)$ and $(V,-\ell)$ that form the basis:

$$\left\{ |H,\ell\rangle, |H,-\ell\rangle, |V,\ell\rangle, |v,-\ell\rangle \right\} \tag{12}$$

for a bipartite two-level quantum system:

$$|\Phi_{pol-OAM}\rangle \in H_{pol} \otimes H_{OAM} \tag{13}$$

The state of X-ray photons emitted near RBH defined by a bipartite two-level system, polarization OAM has the general form:

$$|\Phi_{pol-OAM}\rangle = \alpha_{00}\,|H,\ell\rangle + \alpha_{01}\,|H,-\ell\rangle + \alpha_{10}\,|V,\ell\rangle + \alpha_{11}\,|V,-\ell\rangle \tag{14}$$

with $\sum_{ij}\alpha_{ij}^2 = 1$

IV.

A good measure to characterize the state of the accretion disk photons having acquired the two degrees of freedom (polarization and OAM) is the linear entropy of the system. The linear entropy expresses the degree of mixedness of the system and is related to the density matrix through the relation:

$$S_L = \frac{d}{d-1}(1 - Tr(\rho^2)) \tag{15}$$

where d is the dimension of the Hilbert space of the system, here $d = 4$ for the X-ray photons with two degrees of freedom, and Tr stands for the trace of the density matrix of the composite system.

Fortunately here is not necessary to construct the $4x4$ density matrix of the photons. We determine the state of the system [14], [15] by making a connection with its subsystems components, benefiting from the fact that polarization subsystem is to a certain extent known.

We consider here that a large amount of mixedness present in the subsystems implies a large degree of non-separability of the composite system and vice-versa, a large amount of purity for the subsystems characterize a composite system in mixed state. It is sufficed here to consider only the reduced density matrix of the polarization subsystem to determine the state of the whole system, polarization-OAM.

We do not know anything about the state of the composite system polarization-OAM yet, but the reduced density matrix must fully describe the outcomes of measurements of X-ray photon polarization.

The reduced density matrix of polarization of the bipartite polarization-OAM composite system is found by taking a partial trace over the OAM states:

$$\rho_{red} = Tr_{OAM}(|\Phi_{pol-OAM}\rangle\langle\Phi_{pol_OAM}|) \tag{16}$$

where Tr_{OAM} is the trace over OAM modes of the density matrix on the composite system, the X-ray photon having two degree of freedom polarization and OAM.

The reduced density matrix is the polarization matrix in eq. (11) since two degrees of freedom that compose the bipartite system polarization-OAM refers the same photons. The reduced density matrix yields to:

$$\rho_{red} = \rho_p = \begin{bmatrix} cos^2\chi & 0 \\ 0 & sin^2\chi \end{bmatrix} \tag{17}$$

The linear entropy for the X-ray polarization subsystem considering the reduced density matrix can be written as:

$$S_L = 2(1 - Tr(\rho_p^2)) \tag{18}$$

The reduced density matrix is known and the trace of it is related to the degree of polarization variable according to:

$$Tr(\rho_p^2)) = \frac{1}{2}(1 + \delta^2) \tag{19}$$

The linear entropy for the X-ray photons polarization subsystem takes the simple form [15]:

$$S_L = 1 - \delta^2 \tag{20}$$

The photons emitted by the outer regions of the accretion disk are characterized by horizontal polarization, having higher values of degree of polarization as we have seen earlier, can be considered as being closer to a pure state since the linear entropy is closer to zero, $S_L = 0$. The photons coming from the innermost region vertically polarized with higher degree of polarization are also in a closer to polarization pure state.

We argue base on figure 1 that the minimum of degree of polarization is present for the X-ray photons emitted by the accretion disk in the transition region, at thermal peak energy. The faster block hole is spinning the lower the degree of polarization at the thermal peak. We conclude that at the thermal peak the linear entropy is closer to the maximum value, $S_L = 1$, which induces a state of maximum mixedness in the polarization of photons at the transition region of the accretion disk.

The high degree of mixedness of the photons at the thermal peak determines a high degree of non-separability for the composite system of polarization- OAM considered. The X-ray photons emitted by the RBH accretion disk in the thermal peak are entangled to some extent in polarization and OAM modes.

We have two orthonormal two-level subsystems, polarization and OAM, entangled at the composite system level and the diagonalized reduced density matrix of the polarization system, based on that we would perform Schmidt decomposition over the composite system. Considering for the Schmidt decomposition in the basis (12), the general state of the system in eq. (14) reduces to the elegant form:

$$|\Phi_{pol-OAM}\rangle = \sqrt{\lambda}\,|H\rangle\,|\ell\rangle + \sqrt{1-\lambda}\,|V\rangle\,|-\ell\rangle \tag{21}$$

where λ and $1 - \lambda$ are the Schmidt coefficients and are determined by the eigenvalues of the reduced density matrix of polarization:

$$\lambda = \cos^2\chi \tag{22}$$

$$1 - \lambda = \sin^2\chi \tag{23}$$

With the Schmidt coefficients from eq. (18), the general state of polarization-OAM for the X-ray photons emitted by accretion disk is determined to be of the form:

$$|\Phi_{pol-OAM}\rangle = \cos\chi\,|H\rangle\,|\ell\rangle + \sin\chi\,|V\rangle\,|-\ell\rangle \tag{24}$$

The eq. (19) expresses the non-maximal Bell state for the bipartite system consisting of photons with polarization OAM degrees of freedom. It should be noticed that the degree of entanglement is determined by the polarization angle value.

The non-maximally entanglement expressed in the relations above , for the polarization-OAM degrees of freedom of the X-ray photons emitted in the transition region of the accretion disk transforms in maximally entangle states at the thermal peak where the polarization angle is $\chi = \frac{\pi}{4}$ for an extreme spinning black hole.

$$|\Phi_{pol-OAM}\rangle = \frac{1}{\sqrt{2}}(|H\rangle\,|\ell\rangle + |V\rangle\,|-\ell\rangle) \tag{25}$$

All four Bell states are recovered considering that rotation of polarization angle could acquire both positive and negative values:

$$|\Phi^{\pm}_{pol-OAM}\rangle = \frac{1}{\sqrt{2}}(|H\rangle\,|\ell\rangle \pm |V\rangle\,|-\ell\rangle) \tag{26}$$

$$|\Psi^{\pm}_{pol-OAM}\rangle = \frac{1}{\sqrt{2}}(|H\rangle\,|-\ell\rangle \pm |V\rangle\,|\ell\rangle) \tag{27}$$

V.

Although it may seems premature to speak of the detection and measurement of Bell states since neither polarization, nor OAM modes of X-ray radiation emitted near RBH have not been properly observed to date. The tentative detection of X-ray polarization mission GEMS (Gravity and Extreme Magnetism Small Explorer) that NASA has been scheduled to be launched in 2012 was canceled on budget ground.

The detection and measurement of Bell states are very common in these days optical quantum information processes. Precise setups to measure all four Bell states of single-photons, in polarization and OAM modes, have been reported [20],[21].

The same technical setups could be inferred in measurement of X-ray photons considering the limitations determined by their high energies. However, apparatus required in detection of X-ray Bell states have already been tested in laboratory.

The later improvement in technology applied to X-ray applications such as spiral phase plates, wave plates, beam splitters [24],[25] and leading to quantum computation with X-ray photons [22] were performed well in the laboratory tests.

The idea that once detected by telescopes, X-ray photons emitted by rotating black hole accretion disk at the thermal peak energy could be analyzed and the four Bell states could be measured may not be that exaggerated.

We stated earlier that to detect Bell states of X-ray photons from the accretion disk, we have to specify the exact two-dimensinonal Hermitian space of OAM modes. The most probable OAM modes X-ray photons can acquire, as it can be inferred from the figure 2 are $\ell = \pm 1$, since in the OAM map these two values are the most representative.

Choosing $\ell = \pm 1$ as basis for the OAM modes the X-ray photons Bell states take the form:

$$|\Phi^{\pm}_{pol-OAM}\rangle = \frac{1}{\sqrt{2}}(|H\rangle|1\rangle \pm |V\rangle|-1\rangle) \quad (28)$$

$$|\Psi^{\pm}_{pol-OAM}\rangle = \frac{1}{\sqrt{2}}(|H\rangle|-1\rangle \pm |V\rangle|1\rangle) \quad (29)$$

The setup destined to measure the Bell states of X-ray photons coming from RBH should be calibrated over the states in eq. (23)-(24).

VI.

In conclusion, we argued in the present paper that photons of X-ray radiation emitted by the accretion disk of rotating black holes are entangled in polarization and OAM. The degree of entanglement depends on the region of the accretion disk the X-ray radiation is coming from and the speed BH is spinning.

Photons emitted by accretion disks, influenced by strong gravitational field nearby spinning black holes acquire OAM and suffers rotation of polarization angle. We consider these two degrees of freedom of photons as a bipartite two-level system and infer that the reduced matrix of polarization determined by the degree of polarization is a measure of the mixedness of the system via linear entropy.

We have shown, based on the higher degree of polarization values that X-ray photons emitted in the outer and innermost regions of the accretion disk in near pure states are very weakly entangled. Radiation emitted in the transition region of the disk probe a weak degree of polarization signaling a higher degree of entanglement in polarization and OAM at the composite system level.

The degree of polarization at the transition region weakens as the BH spins faster, fact that allows us to conclude that X-ray photons probe higher degrees of entanglement near faster spinning BH. The X-ray photons maximal entangled states are present for extreme rotating BH.

The maximal entanglement is represented by all four Bell states, deduced via Schmidt decomposition.

Although it may seems premature to speak of measurement of this Bell states, the present development of quantum information apparatus allow us to hope that near future will probe the entanglement in polarization and OAM for X-ray photons emitted near spinning BH.

Detection of Bell states in photons coming from RBH may prove to be important since it suggest that these mysterious astrophysical bodies are capable to implement complex quantum information processes.

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